A NEW CRACK PROPAGATION CRITERION FOR SKIER-TRIGGERED SNOW SLAB AVALANCHES

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ABSTRACT: Dry-snow slab avalanches begin with a local failure in a weak snowpack layer buried below cohesive snow slab layers. If the size of the failed zone exceeds a critical size, rapid crack propagation occurs possibly followed by slab release if the slope is steep enough. The probability to trigger a slab avalanche by a skier or a snowmobile is generally characterized by classical stability indices that do not account for crack propagation. In this study, we propose a new model to evaluate the conditions for the onset of crack propagation in skier-triggered slab avalanches. For a given weak layer, the critical crack length characterizing crack propagation propensity (not considering the additional load by a skier) is compared to the size of the area where the skier-induced stress exceeds the shear strength of the weak layer. The critical crack length is calculated from a recently developed model based on numerical simulations. The skier- induced stress is computed from analytical solutions and finite element simulations to account for slab layering. A detailed sensitivity analysis is performed for simplified snow profiles to characterize the influence of snowpack properties and slab layering on crack propagation propensity. We applied our approach for manually observed snow profiles and compared our results to rutschblock stability tests. Finally, we propose a new stability index for skier-triggered slab avalanches incorporating both failure initiation and crack propagation.

KEYWORDS: Snow avalanche, skier-triggering, failure initiation, crack propagation, PST, slab, weak layer.

1 INTRODUCTION

Skier-triggered slab avalanches (Fig. 1) cause each year more than 100 fatalities in the European Alps. The sound understanding of the stability of the snowpack loaded by a skier or a snowmobile is thus very important.

A slab avalanche can be (remotely) triggered by a skier if the size of the failed zone induced by the additional load of the skier in the weak layer exceeds the critical crack size (Heierli et al., 2011; Schweizer and Camponovo, 2001; Schweizer et al., 2003). If so, rapid crack propagation occurs possibly leading on steep slopes to a slab avalanche.

In general, the skier stability index (e.g., Föhn, 1987b) is a measure to assess failure initiation by a skier. Recently, slab layering which can modify the stress distribution in the snowpack (e.g. due to the so-called bridging effect) was also accounted for in stability metrics (Habermann et al., 2008; Monti et al., 2016; Thumlert et al., 2013; Thumlert and Jamieson, 2014). However, this classical index, which compares weak layer strength to the sum of the shear stress due to the slab load and the additional skier stress does not account for the crucial process of crack propagation.

Fig. 1: Slab avalanche triggered by a snowboarder near Arlberg (Austria). Photo: Remi Petit.

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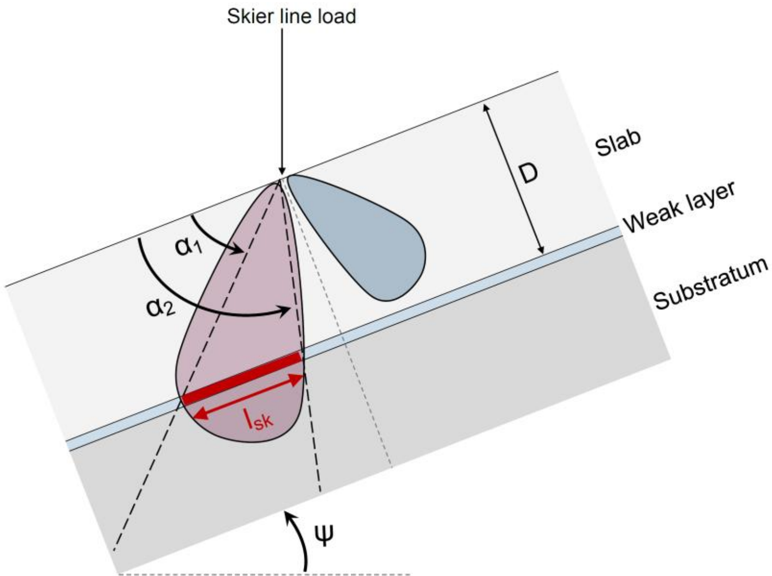

Fig. 2: Schematic of the stresses in the snowpack under the influence of a skier line load.

We propose a new criterion for the onset of crack propagation in a weak snow layer below a cohesive snow slab in presence of an additional line load corresponding to a skier. This criterion compares the length of the area where the stresses induced by the slab and the skier exceed the WL strength to the critical crack length a_c for crack propagation.

The recent model of Gaume et al. (2016) based on discrete element simulations (Gaume et al., 2015) is used to compute the critical crack length a_c. The stress field along the weak layer is computed from analytical solutions (Monti et al., 2016) and finite element simulations to account for slab layering (Habermann et al., 2008; Reuter et al., 2015). By calculating the stress to the WL strength, one can thus determine the length l_{sk} of the area where the stress exceeds the strength and compare it to a_c. A sensitivity analysis is performed for simplified snow profiles to characterize the influence of snowpack properties and slab layering on the skier crack propagation propensity. Additionally, we applied our approach to a field dataset consisting of 251 Rutschblock test results which are completed by a snow pit, snow density and shear frame measurements. Finally, a new stability index is proposed for skier-triggered slab avalanches incorporating both failure initiation and crack propagation.

2 METHODS

2.1 Conditions for crack propagation

We consider a two-dimensional slab-weak layer system. The slab is characterized by its thickness *D*, density ρ, elastic modulus *E*, and Poisson's ratio ν. The weak layer is characterized by its shear strength τ_p, its shear modulus G_{wl} and thickness D_{wl}. Slope angle is denoted by ψ. The onset of crack propagation occurs if $l_{\mathrm{sk}} > a_{\mathrm{c}}$. The skier crack length is obtained by solving:

$$\tau + \Delta\tau > \tau_p \tag{1}$$

where $\tau = \rho g D \sin\psi$ is the shear stress due to the slab weight, $\Delta\tau$ is the additional shear stress due to the skier line load *R* which is defined as (Monti et al., 2016):

$$\Delta\tau = \frac{2R\cos\alpha\sin^2\alpha\sin(\alpha+\psi)}{\pi D} \tag{2}$$

where α is the angle between the snow surface and the line from the skier to the point of interest in the weak layer. Assuming the strength of the weak layer is exceeded along a length l_{sk} within the weak layer, we define two angles α_1 and α_2, locating the edges of this band (of failure). Hence, solving Eq. (1) corresponds to finding the two angles α_1 and α_2 where $\tau + \Delta\tau = \tau_p$ (Fig. 2). Then, the skier crack length l_{sk} can be evaluated by:

$$l_{\mathrm{sk}} = D\left[\frac{1}{\tan\alpha_1} - \frac{1}{\tan\alpha_2}\right] \tag{3}$$

Eq. (1) cannot be solved analytically for $\psi \neq 0$, i.e. if $\tau \neq 0$. It was thus solved using Matlab (fzero function).

Finally, the critical crack length a_{c} is computed using the new formulation proposed by Gaume et al. (2016):

$$a_{\mathrm{c}} = \Lambda\left[\frac{-\tau+\sqrt{\tau^2+2\sigma(\tau_p-\tau)}}{\sigma}\right] \tag{4}$$

in which Λ is a characteristic length of the system associated with the elastic mismatch between the slab and the WL. It is given by:

$$\Lambda = \sqrt{\frac{E' D D_{wl}}{G_{wl}}} \quad (5)$$

with $E' = E/(1-\nu^2)$.

2.2 New stability index

It seems natural to define a new stability index characterizing both initiation and crack propagation propensity:

$$S_p = \frac{a_c}{l_{sk}} \quad (6)$$

Large values of the skier crack length l_{sk} and/or low values of the critical crack length a_c lead to low stability and vice versa.

2.3 Simplified snow profiles and finite element simulations

We calculated the skier crack length for five different typical slab profiles with either a hard or soft substratum (Fig. 3) using the finite element method (FEM), which allows to take into account snowpack layering to evaluate l_{sk}. The FEM model is described in detail in Reuter et al. (2015). The domain is divided into 2-D, quadrilateral plane strain elements with eight nodes each. The mesh was fine enough to avoid mesh size effects. The model was implemented in ANSYS workbench to calculate the skier stress within the weak layer. The layers are considered as linear elastic. The skier load was modeled as a static line load R of 780 N (Schweizer and Camponovo, 2001).

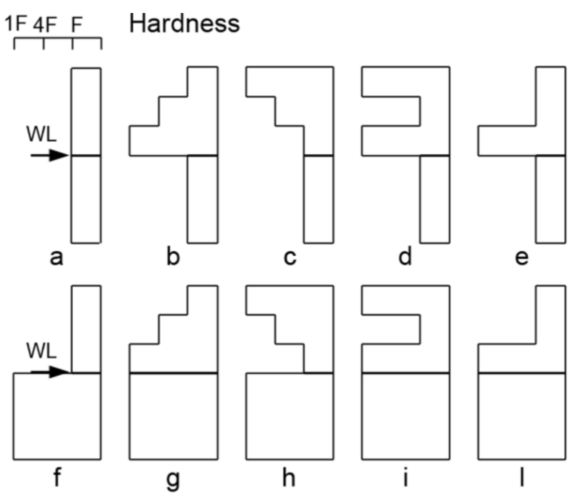

Fig. 3: Ten simplified hardness profiles. The profiles from (a) to (e) have a weak base, while from (f) to (l) a strong base. The arrows highlight the depth where the weak layer was located (not to scale). The simplified profiles have the same characteristics as the ones used by Habermann et al. (2008).

Table 1: Material properties of the layers for the simplified snow profiles.

Layer characteristic	Hand hardness index	Density ρ (kgm^{-3})	Elastic Modulus E (MPa)	Poisson's ratio ν
Soft	F	120	0.3	0.2
Medium	4F	180	1.5	0.2
Hard	1F	270	7.5	0.2
Weak layer	F-	100	0.15	0.2

The simplified profiles have the same characteristics and material properties (Table 1) as those used by Habermann et al. (2008). The values of hand hardness (Fierz et al., 2009) were assigned corresponding to the layer densities (Geldsetzer and Jamieson, 2001). The Poisson's ratio was assumed constant $\nu = 0.2$. The slab layers have a thickness of 0.12 m each, the weak layer of 0.05 m and the substratum of 0.8 m, thick enough to ensure that the results are not influenced by the substratum thickness. As in Habermann et al. (2008) and Monti et al. (2016), the penetration depth of the skier was not taken into account for these calculations. The weak layer shear strength τ_p was assumed equal to 700 Pa.

The critical crack length was evaluated for each profile using Eqs. (4) and (5). For the elastic modulus of the slab, we used the bulk modulus computed using FEM simulations (Reuter et al., 2015), as it accounts for slab layering.

2.4 Field data

The skier crack length l_{sk} and critical crack length a_c were calculated for 160 manually observed snow profiles collected in the Columbia Mountains of western Canada by researchers from the University of Calgary, each including a rutschblock (RB) test (Föhn, 1987a). The tests were all combined with manual snow profile observations, including multiple density measurements of slab and weak layers as well as shear frame measurements of the weak layer shear strength (Jamieson and Johnston, 2001). The elastic modulus of the different layers was derived from manual density measurements using the relation proposed by Scapozza (2004).

The values of a_c and l_{sk} were then compared to the RB score for 3 different cases corresponding to different values of the skier line load R according to Schweizer and Camponovo (2001): (i) skier

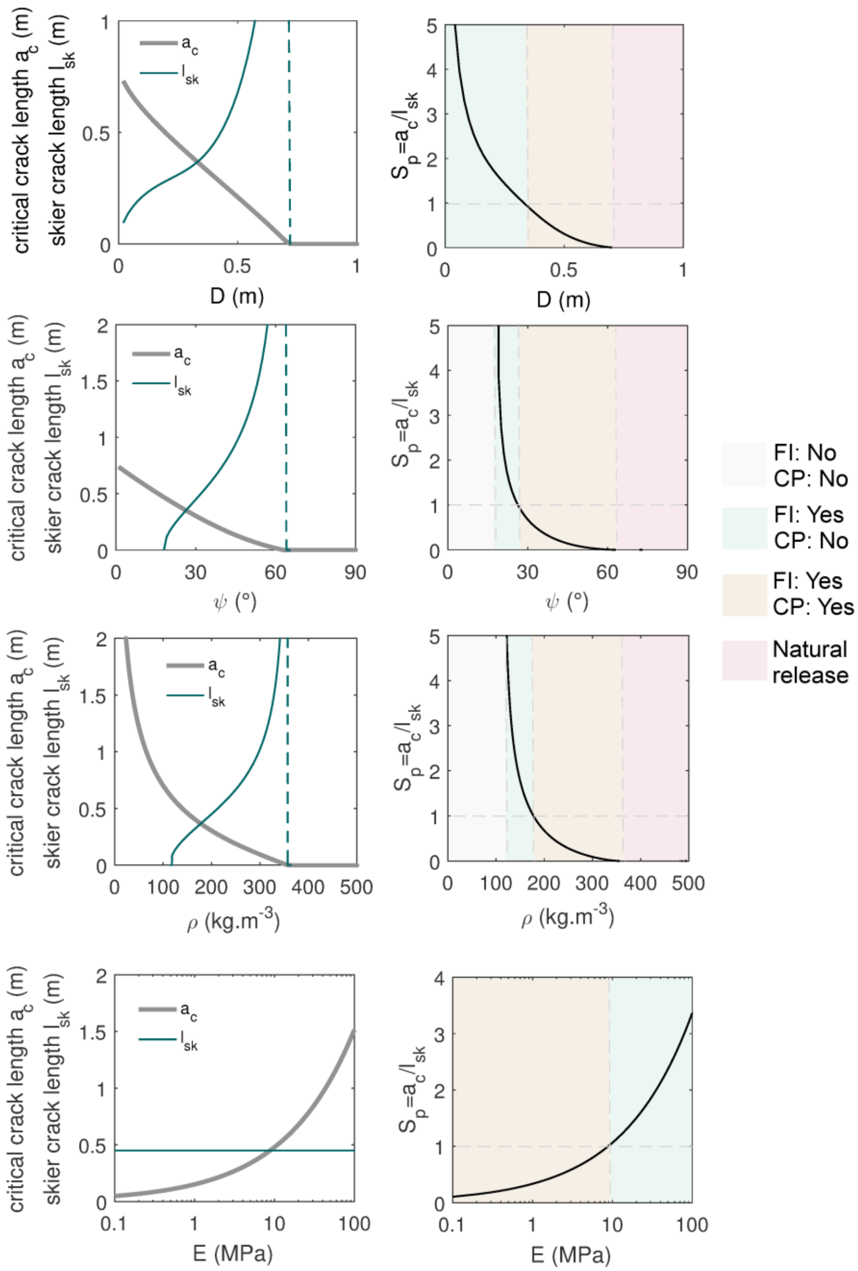

Fig. 4: Sensitivity analysis with independent snowpack properties, i.e. only one parameter was varied while the other properties were kept constant. D=0.4 m, $\psi = 30°$, ρ=200 kg m^{-3}, τ_p = 700 Pa, R=780 N. FI: Failure initiation; CP: Crack propagation.

standing i.e. *R*=780 N; (ii) skier weighting i.e. *R*=1950 N; (iii) skier jumping i.e. *R*=3900 N.

3 RESULTS

3.1 Sensitivity analysis

We performed a sensitivity analysis to assess the effect of snowpack properties on the skier crack length l_{sk}, the critical crack length a_c and propagation stability index S_p. First, only one parameter is varied while keeping all others fixed. Second, to mimic more realistic snow properties, we used empirical formulations to relate (i) slab density to slab thickness, (ii) slab elastic modulus to slab density and (iii) weak layer shear strength to the overlying slab load.

3.1.1 Independent snowpack properties

Fig. 4 shows the results of the sensitivity analysis with independent snowpack properties. As shown in Gaume et al. (2016), the critical crack length a_c decreases with increasing slab thickness *D*, slab density ρ and slope angle ψ but increases with increasing elastic modulus of the slab *E*. On the other hand, the skier crack length l_{sk} increases with increasing slab thickness, slab density and slope angle since the slope-parallel load increases. However, the skier crack length does not change with variations of the elastic modulus because it does not appear in Eq. (1) (Monti et al., 2016), as long as the slab is uniform. As a result of these trends, the propagation stability index S_p decreases with increasing slab thickness, slab density and slope angle but increases with increasing elastic modulus.

From Fig. 4, one can distinguish four different stability regimes: (i) a regime in which neither failure initiation nor crack propagation are possible because $l_{sk} = 0 < a_c$. In this case $S_p \to \infty$; (ii) a regime in which failure initiation is possible but crack propagation cannot occur since $l_{sk} < a_c$, i.e. $S_p > 1$; (iii) a regime in which both failure initiation and crack propagation occur since $l_{sk} > a_c$, i.e. $S_p < 1$ and finally (iv) a regime in which the stress due to the load of the slab on the weak layer is higher than its strength. In this case, the critical crack length is equal to zero and the skier crack length is infinite leading to $S_p = 0$.

3.1.2 Realistic snowpack properties

In general, the elastic modulus of snow is related to snow density (e.g., Camponovo and Schweizer, 2001; Scapozza, 2004; van Herwijnen et al., 2016). In addition, the slab density generally increases with increasing thickness due to settlement processes. This settlement also induces a strengthening of the buried layers. We assume slab density to be related to slab thickness according to $\rho = 100 + 135\, D^{0.4}$ (McClung, 2009) (with an initial density of 100 kg/m^3 for *D*=0) and the shear strength to be related to slab thickness according to $\tau_p = 300 + 1370\, D^{1.3}$. The latter is the same expression as found by Bazant et al. (2003) and McClung (2003) with a cohesion *c*=300 Pa. Finally, we assume the elastic modulus of the slab to be related to density according to a power law fit to the data of Scapozza (2004): $E = 5.07 \times 10^9 (\rho/\rho_{ice})^{5.13}$ with $\rho_{ice} = 917$ kg/m^3.

Using these relationships, the stability behavior is significantly different (Fig. 5) than for the sensitivity analysis with independently varying properties. In this case, the critical crack length is almost independent of slab thickness as both the elastic modulus of the slab and the strength of the weak layer increase, counteracting the increase in load. Furthermore, the skier crack length first increases strongly until D≈0.3 m and then decreases. The skier crack length is higher than the critical crack length for thin snowpacks (0.2 m < D < 0.5 m) and lower for thicker ones (D>0.5 m). As a consequence, the propagation stability index is below 1 for D<0.5 m and crack propagation is possible. Above this threshold, the weak layer is located too deep to be triggered. This result is in line with previous field observation and measurements (e.g., Schweizer and Jamieson, 2001; van Herwijnen and Jamieson, 2007). Note however that very shallow slabs generally correspond to very low slab densities (excluding wind slabs) for which slab fractures might occur before the onset of crack propagation (Gaume et al., 2015, 2015b; Schweizer et al., 2014).

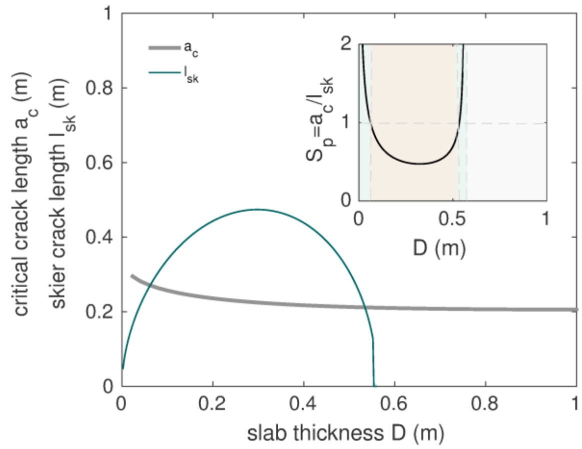

Fig. 5: Sensitivity analysis for realistic snowpack properties, i.e. WL shear strength, slab thickness and slab density are interrelated for $\psi = 30°$.

3.2 Influence of slab layering

In general profiles with a hard substratum have a higher skier crack length, lower critical crack length and thus lower propagation stability index compared to their corresponding profile with softer substratum (Fig. 6).This is in line with the empirical results shown in van Herwijnen and Jamieson (2007) who observed an increase in skier-triggering probability for harder substrata and with results from the finite element simulations of Habermann et al. (2008). Furthermore, profiles for which a hard slab is close to the WL have a lower initiation and propagation propensity (higher S_p) than those with a hard slab close to the snow surface. Note however that for this theoretical analysis, the skier penetration depth was not accounted

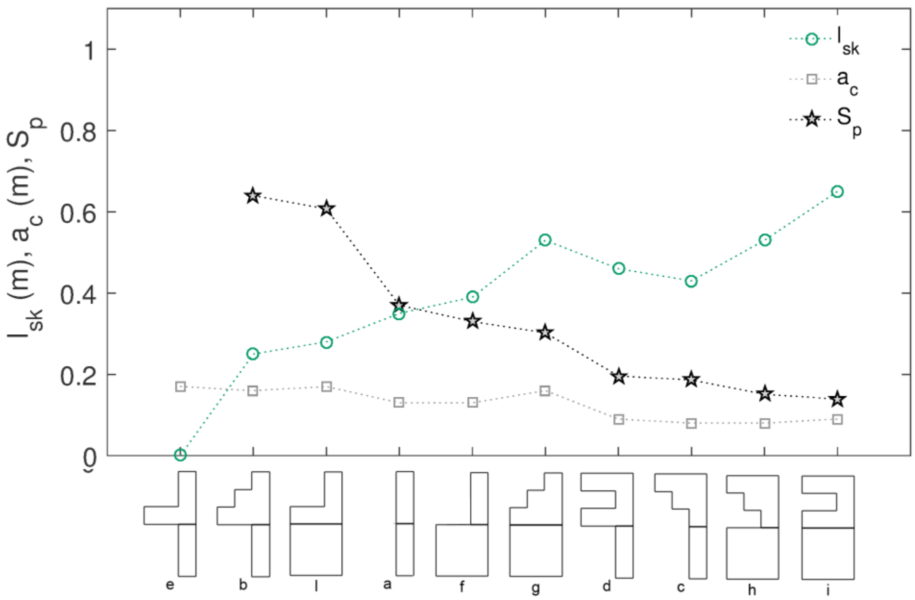

Fig. 6: Skier crack length, critical crack length and skier propagation index as a function of the profile type sorted to obtain a decreasing trend of S_p.

for, but is likely to reduce the propagation stability index of profiles with a soft slab close to the snow surface.

3.3 Comparison with field data

For this analysis, we denoted the stability as “poor” for RB scores of 1 and 2,“fair” for RB scores of 3 and 4, and “good” for RB scores of 5, 6 and 7 similar to Monti et al. (2016). The observed stability based on RB scores correlates very well with the critical crack length (Fig. 7c, no intersecting notches). The critical crack length increases from approximately 0.5 m to 1.2 m for observed stability increasing from poor to good. The skier crack length was computed for different cases: (i) skier standing i.e. R=780 N; (ii) skier weighting i.e. R=1950 N; (ii) skier jumping i.e. R=3900 N. The analysis revealed that the crack length computed for the case of a skier jumping correlates best to the observed stability (RB score). Indeed, for most snow covers, the skier crack length was zero when using the additional load corresponding to a skier standing. Hence, R=3900 N was used to compare the skier crack length to Rutschblock data. Fig. 7a shows that the skier crack length decreases from 0.5 m to zero for observed stability increasing from poor to good (i.e. RB score increasing from 1 to 7). It can be also noticed on Fig. 7b that the skier crack length decreases with increasing critical crack length and both quantities correlate well. From this analysis, one can remark that the criterion $l_{\mathrm{sk}} > a_c$ i.e. $S_p < 1$ does not exclude all data points with a poor stability. In fact, in this case a criterion $l_{\mathrm{sk}} > a_c/3$ i.e. $S_p < 3$ would perform better which is very likely due to the fact that we chose the jumping case to compute l_{sk} to avoid too much nil values. Typically, poor stability corresponds to $l_{\mathrm{sk}} > 0.25$ m and $a_c < 0.75$ m which leads to $S_p < 3$. Finally, both the skier crack length and critical crack length discriminate well between the stability classes (poor, fair, good) since the notches in the boxplots do not overlap, which indicates, with 95% confidence, that the true medians differ.

4 DISCUSSION AND CONCLUSIONS

We developed a new model to describe snow stability in terms of weak layer strength and critical crack length for crack propagation. We compare the crack length due to a skier, i.e. the length at depth of the WL for which the shear stress due to the slab and the skier exceeds the shear strength of the WL, to the critical crack length. The critical crack length is computed from a recent model based on discrete element simulations (Gaume et al., 2016).

A detailed sensitivity analysis was performed to study the effect of each snowpack properties. For realistic values of the system parameters, we showed that the skier initiation and propagation propensity first increases with increasing slab thickness and then decreases for a thickness of about 0.5 m. This threshold depends on snowpack properties, in particular, it should increase with

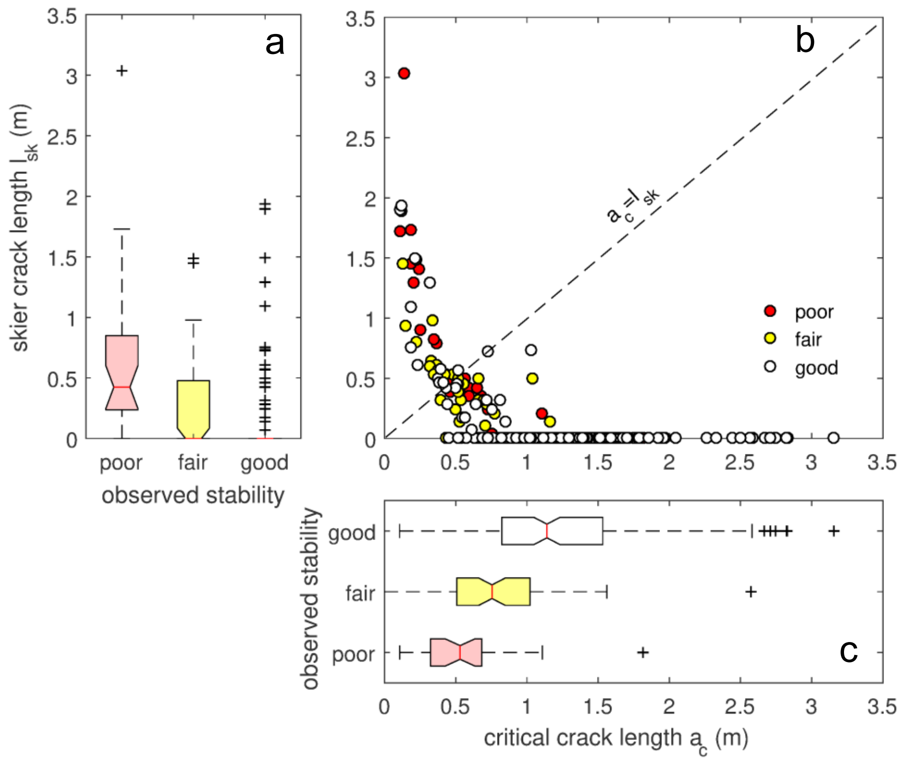

Fig. 7: Scatter plot and boxplots of the observed stability (based on RB scores) vs critical crack length a_c and skier crack length l_{sk} for R=3900 N.

increasing slope angle since a_c decreases and l_{sk} increases with increasing ψ.

The effect of snowpack layering on the skier crack length, critical crack length and skier propagation index was quantified by means of finite element simulations with a linear-elastic assumption. It revealed that snowpacks with hard substrata result in lower values of the skier propagation index compared to softer substrata. This is due to higher stress concentrations at the depth of the WL and is in line with empirical field evidence (van Herwijnen and Jamieson, 2007). In addition, it was shown that snowpacks with hard slab layers adjacent to the WL were less prone to initiation and crack propagation than snowpacks with hard slab layers close to the snow surface. Skier penetration depth was not considered but will very likely influence the presented results, in particular for soft slabs.

Finally, our new skier propagation criterion was applied to manually observed snow profiles and compared to the RB score. The skier crack length obtained for a skier jumping and the critical crack length correlate well with the observed stability, i.e. the RB score, confirming the usefulness of the proposed approach to refine current stability estimates.

In the future, we plan to include the skier penetration depth in the model as well as the possible compressive failure of the weak layer using the Mohr-Coulomb-Cap model developed by Reiweger et al., 2015.

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