

FRACTURE ENERGY OF WEAK SNOWPACK LAYERS

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**ABSTRACT:** Recent experimental and theoretical work has provided new insight into weak layer fracture in view of dry-snow slab avalanche release. The two main parameters which influence this process are slab stiffness (i.e. elastic modulus) and weak layer fracture energy. These parameters are therefore of importance for avalanche forecasting. So far, only few values of specific fracture energy exist, mainly because the stiffness of the slab cannot easily be determined. We performed about 150 propagation saw tests to calculate weak layer fracture energy. To estimate the stiffness of the slab that is required for the calculation, we applied two methods: (1) in-situ penetration resistance measurements, and (2) video imaging of the fracture process. Both methods provided values of the average slab modulus on the order of a few MPa. The resulting weak layer fracture energies ranged between  $0.28$  and  $2.2 \text{ J m}^{-2}$  – considerably higher than previously published values. Many more measurements are needed for a comprehensive dataset of weak layer fracture energy.

1. INTRODUCTION

Recent experimental and theoretical work suggests that the collapse of the weak layer plays an important role in the release of dry-snow slab avalanches (Heierli et al., 2008; van Herwijnen et al., 2010). As the weak layer fractures it collapses releasing gravitational potential energy, which is used to drive the fracture forward. The energy released from this collapse is generally more than sufficient to overcome the weak layer fracture energy (Heierli et al., 2010). However, before a fracture can propagate through the weak layer, an initial fracture has to grow to a critical size. The two main parameters which influence this latter process are slab stiffness (i.e. elastic modulus) and weak layer fracture energy.

Sigrist and Schweizer (2007) pointed out that the energy that has to be exceeded to fracture a weak layer depends on the material properties of the weak layer, whereas the energy that is available for crack propagation depends mainly on the material properties of the overlaying slab and the slope normal collapse height of the weak layer. They also provided the first values of fracture energy derived from field measurements. Using a similar design for the field test as Gauthier and Jamieson (2006) to determine the critical cut length, they used a finite element (FE) model to

determine the weak layer fracture energy. The elastic modulus of the slab that is required to calculate the fracture energy was derived from in-situ penetration resistance measurements using a snow micro-penetrometer (SMP; Schneebeli and Johnson, 1998). For a persistent weak layer consisting of mainly faceted crystals and some depth hoar (1-2 mm in size) a critical energy release rate (or weak layer fracture energy) of  $0.07 \pm 0.02 \text{ J m}^{-2}$  was found. Similarly low values for the critical energy release rate have been reported in a number of preceding – mainly laboratory – studies (e.g. Kirchner et al., 2000; Schweizer et al., 2004; Sigrist et al., 2006).

Assuming a homogeneous slab layer Sigrist (2006) provided an approximate analytical solution for determining the critical energy release rate by only considering the slope normal displacement. A more comprehensive analytical solution which considers all terms contributing to the mechanical energy was derived by Heierli (2008).

The above reported low values of the specific fracture energy have never been confirmed by an independent method. In any case, the values are highly sensitive to the assumptions made when the elastic modulus of the slab is determined. Furthermore, thus far there is no dataset with values of the specific fracture energy for various types of weak layers.

Recently, van Herwijnen and Heierli (2010a,b) proposed an alternative method for determining the critical fracture energy. They analyzed the deformation field as obtained from a video sequence of the fracture experiments with particle image velocimetry (PIV). From the measured amount of bending they derived an average elastic modulus for the slab, and independently thereof the weak layer fracture energy. This method,

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which we will call PIV method, allows one to verify the values of the modulus derived from the SMP signal.

The aim of the present study is to provide additional values of specific fracture energy for different types of weak layers based on field experiments using the propagation saw test. Based on an improved geometry for the FE model and a recently developed algorithm to extract the modulus from the SMP signal (Marshall and Johnson, 2009), we will provide new values for the fracture energy of weak layers determined with the SMP-FE method, and finally compare those with results obtained with the PIV method.

## 2. METHODS

During the winter 2009-2010 we have performed propagation saw tests mainly following the procedure as described in Gauthier and Jamieson (2006). The block length was always at least 120 cm, the cut direction was always up-slope and the top and bottom end faces of the blocks were cut slope perpendicular. Measurements included the critical cut length, slab thickness and slope angle. A nearby snow profile provided slab and weak layer stratigraphy and layer density. All tests were completed with snow micro-penetrometer measurements. In one occasion we also recorded a video sequence of the fracture tests (Figure 1).

To determine the specific fracture energy using the FE method or the analytical solution, the elastic properties of the slab need to be derived. Three methods can be applied to estimate those: (1) The modulus can be estimated based on density using a relation such as provided by Scapozza (2004) or Sigrist (2006). (2) The modulus can be evaluated from the SMP signal using an algorithm proposed by Johnson and Schneebeli (1999) and recently improved by Marshall and Johnson (2009); we will call this method the SMP method. (3) The modulus can be determined using the PIV method (Figure 1) as suggested by van Herwijnen and Heierli (2010a).

When determining the elastic modulus of the slab using the SMP method, the micro-modulus was obtained with the algorithm described by Marshall and Johnson (2009), i.e. we did not use a factor to fit the micro-modulus to observed values of the Young's modulus as previously done by Kronholm (2004) or Sigrist (2006). Furthermore, we only used quality checked SMP signals for the analysis; signals that exhibited a significant drift or signs of a frozen sensor were discarded.

Once the elastic properties are estimated the specific fracture energy of the weak layer can be

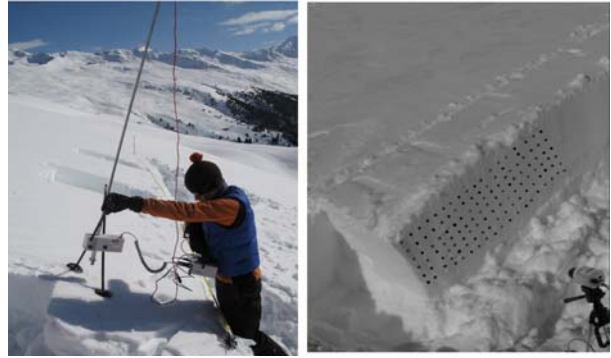


Figure 1: The elastic properties of the slab were obtained from either in-situ SMP measurements (left) or a video sequence of a propagation saw test (right).

evaluated by either (a) using the analytical solution provided by Heierli (2008, Eq. 4.13), or (b) using a finite element model as described by Sigrist (2006) and Sigrist and Schweizer (2007). With the finite element method the slab can consist of several layers with varying material properties, whereas an average modulus is used with the analytical solution. We adapted the geometry of Sigrist's (2006) FE model to account for up-slope sawing and for perpendicular front and end faces. Also we used a thicker base (substratum) layer (40 cm) than Sigrist (2006). A preliminary sensitivity analysis revealed that up to a thickness of about 40 cm the thickness of the base layer considerably influenced the calculated fracture energy values.

## 3. RESULTS

We had reliable SMP signals to derive the elastic properties of the slab on 8 field days. For one of these days (12 March 2010) we also recorded a video sequence of one experiment. On each day we performed between 14 and 24 (median: 18) fracture tests on the same weak layer in combination with SMP measurements.

Table 1: Summary statistics for field tests

Parameter	Median	Min	Max
Slope angle (°)	30	20	33
Slab thickness (cm)	37	25	43
Slab density (kg m <sup>-3</sup> )	250	170	280
Critical cut length (cm)	28	15	43
Elastic modulus (MPa)	3.4	1.8	12
Specific fracture energy (SMP-FE method) (J m <sup>-2</sup> )	1	0.28	2.2
Specific fracture energy (analytical solution) (J m <sup>-2</sup> )	0.15	0.06	0.32

The weak layers we tested mainly consisted of rounding faceted particles (FCxr) and faceted crystals (FC); typical crystal size was 1-2 mm. Slab thickness (measured slope perpendicular) ranged from 25 to 43 cm with a median of 37 cm; slope angle was between 20 and 33° with a median of 30° (Table 1). Mean slab density varied between 170 and 280 kg m<sup>-3</sup> (median: 250 kg m<sup>-3</sup>). With 28 cm the median critical crack length was smaller than the median slab thickness.

The values for the average (weighted by layer thickness) elastic modulus of the slab layers obtained with the SMP method were in the range of 1.8 to 12 MPa (median: 3.4 MPa). For 12 March 2010 the mean slab stiffness determined with the SMP method was 3.8 ± 0.8 MPa while with the PIV method the elastic modulus of the slab was 1.3-1.6 MPa depending on the calculation method (van Herwijnen and Heierli, 2010a).

The median specific fracture energy calculated with the FE model varied between 0.28 and 2.2 J m<sup>-2</sup> with a median value of 1 J m<sup>-2</sup>. The values obtained with the analytical solution using the mean slab modulus were considerably smaller with a median value of about 0.15 J m<sup>-2</sup>. van Herwijnen and Heierli (2010a) evaluated the specific fracture energy for the weak layer tested on 12 March 2010 using the PIV method to 1.4 J m<sup>-2</sup>, whereas the SMP-FE method yielded a mean value of 2.1 ± 0.8 J m<sup>-2</sup> (*N* = 23).

#### 4. DISCUSSION

The values of weak layer fracture energy obtained with the FE model using the SMP method to estimate the moduli were about an order of magnitude larger than the single value reported by Sigrist and Schweizer (2007). The difference seems – in part – to be related to the fact that Sigrist (2006) adjusted his SMP micro-modulus to values of the modulus obtained with a dynamic measuring method at 100 Hz. Accordingly, for our median slab density (250 kg m<sup>-3</sup>) a modulus of about 21 MPa would result. Our median slab stiffness was 3.4 MPa with a slab density quite a bit higher than the slab density reported by Sigrist and Schweizer (2007). Obviously, using smaller values for the modulus increases the specific fracture energy as more mechanical energy is available due to increased bending.

For the one day when both methods worked, the values of the modulus obtained with the two different methods are in reasonable agreement. The elastic modulus obtained with the PIV method is somewhat lower than that obtained with the SMP method. This suggests that the order of

magnitude of these values is correct and that the values used by Sigrist and Schweizer (2007) are too high. The weak layer fracture energies for that day obtained with the two independent methods were in good agreement.

On all field days, the slab layers contained some hard layers or crusts. Such stiff layers can substantially reduce the amount of bending so that one would expect the analytical solution which uses an average modulus to yield higher specific fracture energy than obtained with the FE method. Nevertheless, the values calculated with the analytical solutions were considerably smaller. This is most likely due to the fact that for the analytical solution the substratum, as well as the weak layer, are assumed to be infinitely stiff. Deformation of the slab is therefore solely contained in the section of the beam which is not supported anymore (i.e. up to the crack tip). The FE simulations, as well as the measurements presented in van Herwijnen and Heierli (2010a) clearly show that the slab deforms ahead of the crack tip, up to a distance roughly equal to the crack length. Therefore, the increased deformation results in more energy being released and a higher specific fracture energy for the weak layer is obtained.

As we only tested a few weak layers we cannot make any conclusion on the dependence of the specific fracture energy on weak layer properties. We only observed that weak layers with low specific fracture energy tended to fail at low rutschblock scores, and vice versa.

#### 5. CONCLUSIONS

We have performed about 150 propagation saw tests to obtain the weak layer fracture energy from measurements of critical crack length, slab thickness and density. To estimate the stiffness of the slab we have applied two methods. One method is based on analyzing the SMP signal, the other one does not need sophisticated instrumentation but the stiffness is derived from bending of the slab using video images. For the one day when both methods worked the modulus obtained with both methods was in reasonable agreement.

The specific fracture energies calculated with the FE method for our weak layer/slab configurations were about 1 J m<sup>-2</sup>. These values are large compared to the only previously published value of 0.07 J m<sup>-2</sup>. The reason for the discrepancy is not entirely clear. However, we have tested our FE model with the input data previously published by Sigrist and Schweizer (2007) and were able to reproduce their low value. In addition, with the PIV method, an independent approach, a similarly high

value of the fracture energy for the weak layer tested on 12 March 2010 was obtained. Accordingly, we are confident that the values of weak layer fracture energy we calculated are realistic.

For the future it will be essential to perform many more measurements in order to obtain a comprehensive dataset of weak layer fracture energy. Eventually, this will allow numerical snow cover models to provide critical cut lengths for simulated weak layer/slab configurations as a measure of instability.

#### ACKNOWLEDGEMENTS

For help with the field work we would like to thank Susanna Hoinkes, Christoph Mitterer and Jake Turner.

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