First edition, 8 September 2016

Naghmeh Nazamia, Christian Schwickb,c, Miroslav Kopecky, Tomas Soukupd, Erika Orlitovad, Felix Kienastc, Jochen A.G. Jaegera,e

a Concordia University Montreal, Department of Geography, Planning and Environment, 1455 De Maisonneuve Blvd. West, Suite H1255, Montreal, QC, H3G 1M8, Canada
b Die Geographen schwick+spichtig, Turbinenstrasse 60, CH-8005 Zurich, Switzerland
c Swiss Federal Research Institute WSL, Zürcherstrasse 111, CH-8903 Birmensdorf, Switzerland
d GISAT and European Topic Centre on Urban, Land and Soil Systems (ETC-ULS) of the European Environment Agency, Prague, Czech Republic
e Primary contact, jochen.jaeger@concordia.ca

The Urban Sprawl Metrics (USM) Toolset is a geographic information system (GIS) toolset and was developed using Python and C+ languages. This tool is freely available under the Creative Commons Licence1 and can be downloaded from the Swiss Federal Institute of Forest, Snow and Landscape Research (WSL) homepage (www.wsl.ch/zersiedelung).

Fig. 1: Example of a landscape from Switzerland that includes built-up areas (close to Zurich). The USM Toolset can be used to measure the degree of urban sprawl of this landscape (photo: J. Jaeger, 2015).

1 distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (http://creativecommons.org/licenses/by-nc-nd/4.0/), which permits non-commercial re-use, distribution, and reproduction in any medium, provided the original work is properly cited, and is not altered, transformed, or built upon in any way. Permission is granted subject to the terms of the License under which the work was published. Please check the License conditions for the work which you wish to reuse.
1. Introduction

The USM Toolset was developed to facilitate the calculation of Weighted Urban Proliferation (WUP) and all components of urban sprawl for landscapes that include built-up areas (e.g., dispersion (DIS), land uptake per person (LUP); Fig. 1). The Toolset is straightforward to use. The language of the user interface is English. The Toolset requires three input data:

1. the binary map of built-up areas (settlements areas and/or solitary buildings), in the ESRI raster format;
2. the map of reporting unit(s) (e.g., municipalities, districts, or a grid of a certain cell size) in geodatabase feature class or shapefile format; and
3. the number of inhabitants and jobs for the reporting unit(s) (this information has to be saved by the user in the attribute table of the reporting unit(s) shapefile).

2. Important background information

The metric of Weighted Urban Proliferation (WUP) has three components: PBA, DIS and LUP (or UD) (Fig. 2).

![Diagram](https://via.placeholder.com/150)

**Fig. 2:** The relationships between the WUP metric and its components DIS, PBA, and LUP (EEA & FOEN 2016: 39). The DIS, PBA and UD (= 1/LUP) metrics are intensive metrics. $A_{\text{reporting unit}}$ = area of the reporting unit (the landscape studied); $A_{\text{built-up}}$ = size of built-up area in the reporting unit; $N_{\text{inh/jobs}}$ = number of inhabitants and jobs in the built-up area of the reporting unit. The shapes of the weighting functions are shown in the boxes as indicated.

Users who already have sufficient knowledge of the definition of built-up areas and the metrics of urban sprawl can continue reading in section 3 (installation of the USM Toolset). However, if the users do not yet have adequate background knowledge, we highly recommend that they read this section carefully or the paper by Jaeger and Schwick (2014) or the first part (sections 2.1 and 2.2) of Chapter 2 "Measurement of urban sprawl, base data, and hypotheses about potential drivers" in the report "Urban sprawl in Europe" (EEA & FOEN 2016) for more detailed information (Fig. 3), e.g., about the meaning of the values of WUP and DIS.
2.1 Definition of built-up areas

Built-up areas “may include various types of settlement and buildings, ranging from places with urban character to villages to separate single buildings in the open landscape. Generally, a built-up area is defined as a surface covered by man-made structures. Roads and railways outside towns and cities are not included in this definition, since they are not perceived to be part of urban sprawl (but rather contribute to landscape fragmentation)” (EEA and FOEN, 2016, p. 47).

For the purpose of comparisons between different regions (or for one region between different points in time), the definition of the built-up areas must be chosen in a precise and consistent way. For smaller regions, usually there are more detailed datasets on ‘built-up areas’ available (e.g., data on the elements of urban surface such as building footprints). However, for large areas, data on built-up areas do not usually include such details of the urban surface. It should be noted that for a meaningful comparison between different points in time, it is necessary to use the same delineation criteria of built-up area. Examples are given in Nazarnia et al. (2016).

2.2 Metrics of urban sprawl

Weighted Urban Proliferation (WUP) has three components: PBA, DIS and LUP (or UD) (Fig. 2). In addition, the two metrics of TS and UP are defined here.

The proportion of built-up areas (PBA) is the proportion of the size of built-up areas to the size of the landscape (reporting unit): PBA = Area of built-up area / Area of reporting unit.

Degree of urban dispersion (DIS) measures the dispersion of built-up areas based on the distances between any two points within the built-up areas (Jaeger et al. 2010b). DIS is expressed in urban permeation units per
Square meter of built-up area (UPU/m²). The more dispersed the built-up areas, the larger the value of DIS. Therefore, more compact built-up areas have lower values of DIS than more dispersed built-up areas.

$w_1(\text{DIS})$ is a weighting function for DIS which assumes values between 0.5 and 1.5 to give higher weights to the more dispersed built-up areas and lower weights to less dispersed areas (Jaeger and Schwick 2014).

**Total Sprawl (TS)** is defined as the average sum of the weighted distances between all points in the urban area and randomly chosen second points where each second point is not farther away from the first point than the horizon of perception (HP). The value of TS is the product of DIS and the total amount of built-up area ($TS = \text{DIS} \times \text{Area of built-up area}$). To learn more about TS, see Jaeger et al. (2010b).

**Utilization Density (UD)** measures the number of people living and working per km² of built-up area. The more people and jobs are located in a built-up area, the higher the land utilization as measured by utilization density (UD). This metric is expressed in inhabitants and jobs per square kilometer of built-up areas (inhabitants+jobs / km²).

$w_2(\text{UD})$ is a weighting function for UD which assumes values between 0 and 1 to give lower weights to more intensively utilized urban areas, i.e., those that have more inhabitants and jobs. The value of $w_2(\text{UD})$ is close to 1 when there are less than 40, and close to 0 when there are more than 100 inhabitants and jobs per hectare of built-up area (Jaeger and Schwick 2014).

**Land Uptake per person (LUP)** is the area of land that is used per inhabitant or job within the built-up areas and expressed in square meters per inhabitant or job (m²/inh. or job). ($LUP = \text{Area of built-up areas}/\text{Number of inhabitants and jobs}$). High LUP values indicate that more space is used per inhabitant or workplace compared to areas where LUP values are lower. LUP is in fact the reciprocal of UD: $LUP = 1/UD$.

**Urban Permeation (UP)** is a measure of the permeation of a landscape by built-up areas. It accounts for the DIS and PBA and is expressed in urban permeation units per m² of landscape (UPU/m²): $UP = \text{PBA} \cdot \text{DIS}$.

**Weighted Urban Proliferation (WUP)** is the main metric used to quantify urban sprawl. It is the product of the Urban Permeation (UP), the weighting of DIS ($w_1(\text{DIS})$) and the weighting of the UD ($w_2(\text{UD})$). WUP is expressed in urban permeation units per square meter of landscape (UPU/m²): $WUP = UP \cdot w_1(\text{DIS}) \cdot w_2(\text{UP})$. More detailed information about these metrics of urban sprawl can be found in Jaeger and Schwick (2014), and in Jaeger et al. (2010b, p. 431, Fig. 4) regarding the cross-boundary connections (CBC) procedure.

### 2.3 Choice of the Horizon of Perception

Calculation of the dispersion of built-up areas (DIS) and Weighted Urban proliferation (WUP) requires a defined scale of analysis, which is specified by the Horizon of Perception (HP). The user can choose the size of the HP between 0.2 and 10 km. However, the default value of HP in the USM Toolset is 2 km, and the weighting function for the computation of weighted Dispersion ($w_1(\text{DIS})$) operates properly only when 2 km is selected. The reason is that the weighting of DIS as a component of WUP was chosen for this scale of analysis of urban sprawl based on expert opinion (see Jaeger and Schwick 2014 for details). If users are interested in using a different value of HP they may need to consider modifying the weighting function equation accordingly in the sivalues.exe tool (see section 3.1). However, working on the logic of suitable weighting functions for HPS other than 2 km should be done in a cautious way and this remains future work.

### 3. Installation of the Urban Sprawl Metrics (USM) Toolset

The Urban Sprawl Metrics Toolset works with ArcGIS version 10.1 (ESRI, 2010) or higher. Only an ArccInfo license of the ArcGIS software is required for the installation of this Toolset (no additional ArcGIS extensions are needed). Minimum requirements for the system (PC/laptop) on which the Toolset will be installed are:

- 1) 4 GB or more Random Access Memory (RAM).
3.1 Urban Sprawl Metrics Toolset archive

The USM Toolset is distributed as a “zip” archive called “USM_Toolset.zip”. The toolset can be installed from http://www.wsl.ch/info/fokus/zersiedelung/index_DE.

The USM Toolset archive contains five files:

1. “USM_toolset.tbx” which is the tool that installs in ArcGIS.
2. “1_Si_value.py” which is the Python script for the computation of Si values (see section 4.1 for explanation of Si values).
3. “2_metrics.py” which is the Python script for computation of the urban sprawl metrics (e.g., DIS, UP, WUP) (see section 2.2 for information about urban sprawl metrics).
4. “3_cleaning.py” which is the Python script for performing directory clean-ups, and finally
5. “sivalues.exe” which is a tool that is being used along with 1_Si_value.py script for the computation of Si values.

3.2 Step by step installation guide

**Step 1:** Download the “USM_toolset.zip” archive from http://www.wsl.ch/info/fokus/zersiedelung/index_DE.

**Step 2:** Extract (unzip) the “USM_Toolset.zip” archive into a folder where the tools will be installed. (Note that you need 10 GB or more free space on the disc where you locate this folder.)

**Step 3:** Open the ArcMap window (you need to have the updated license of this component of ArcGIS suite).

**Step 4:** Open the ArcToolbox window (Fig. 4).

![ArcToolbox in Arcmap.](image)
Step 5: Right-click (use right button of the mouse) on the ArcToolbox icon and select “Add Toolbox” from the popup menu (Fig. 5).

Fig. 5: ArcToolbox-Add toolbox.

Step 6: From the newly opened window, skip to the folder where the “USM_Toolset.zip” was stored and unzipped. Select USM_toolbox.tbx file and click “Open” (Fig. 6).

Fig. 6: Add toolbox-USM_toolset.tbx.
After completing step 6, a new toolset called “USM Toolset” is added to the ArcToolbox window. This toolset contains three tools: “1-Si values calculation”, “2-Metrics calculation” and “3-Cleaning” (Fig. 7).

![Fig. 7: USM Toolset added to the ArcToolbox.](image)

**Step 7:** Right-click (use right button of the mouse) on the “1-Si values calculation” icon and select properties from the popup menu.

**Step 8:** From the newly opened window, go to the “Source” tab and then from the “Script File” bar skip to the folder where the USM_Toolset archive is stored and select “1_Si_value.py” and click on the ‘OK’ button (Fig. 8). Keep all the default properties as is and do not make any changes.

![Fig. 8: 1-Si values calculation properties window.](image)
**Step 9:** Repeat steps 7 and 8 for the next tool (2-Metrics calculation). Right-click (use right button of the mouse) on the “2-Metrics calculation” icon and select properties from the popup menu. From the newly opened window go to the “Source” tab and then from the “Script File” bar skip to the folder where the USM_Toolset archive is stored, select “2_metrics.py”, and click on the ‘OK’ button.

**Step 10:** Repeat steps 7 and 8 for the third tool (3-Cleaning). Right-click (use right button of the mouse) on the “3-Cleaning” icon and select properties from the popup menu. From the newly opened window go to the “Source” tab and then from the “Script File” bar skip to the folder where the USM_Toolset archive is stored, select “3_cleaning.py”, and click on the ‘OK’ button.

In order to avoid the need for repeating the implementation of above-mentioned steps, the user should save the ArcMap project after completing the final step (step 10). The USM Toolset will be automatically available whenever the saved ArcMap project is being used.

**4. How to use the Urban Sprawl Metrics Toolset**

In the following sections, a step-by-step guide to use the USM Toolset is described. Users should consider preparing their input data and working directories before using the USM Toolset.

Users need to have two working folders: (1) a ‘Directory’ folder and (2) an ‘Output’ folder. In the directory folder users should store their input data along with the sivalues.exe Tool. The three components of the directory folder should be (1) the binary map of built-up areas2, (2) geodatabase feature class or Shapefile of the reporting unit(s)/area of study, and (3) sivalues.exe file. Users should keep the Output folder empty because the outputs of the calculations will be sorted in this folder automatically.

**4.1 Si values calculation tool**

The purpose of this tool is to calculate the Si values for each pixel of urban area. The metrics of urban sprawl characterize sprawl in a geometric perspective, and their calculation is based on all distances between any two points within the urban area. The so-called Si values are in fact the mean of the weighted distances between any pixel of urban area and all other urban pixels within the horizon of perception.

The input of the tool is the binary map of built-up areas (settlements areas and/or solitary buildings) in ESRI raster format. The binary raster has two values (0 values for non-built-up areas and 1 value for built-up areas). At this stage, the user should choose a value for the horizon of perception. The default HP of the USM Toolset is 2 km and the calculation of metrics of urban sprawl is based on weighting functions that are appropriate for a horizon of perception of 2 km. So it is highly recommended that users keep the default value (HP = 2000 m).

**4.1.1 How to use the Si values calculation tool**

1. Open the saved ArcMap project in which the USM Toolset has been installed. From Arctoolbox, select USM toolset and click on the first tool (1-Si values calculation) (Fig. 9).
2. From the ‘Path to the sivalues.exe Tool’ bar, skip to the ‘Directory’ folder and click on the ‘Add’ button.
3. From the ‘Input Raster of the Built-up Areas’ bar, skip to the directory folder, select the binary map of built-up areas and click on the ‘Add’ button.
4. In the ‘Horizon of Perception’ bar, keep the default value of 2000 m.
5. Finally from the ‘Output Directory’ bar, skip to the ‘Output’ directory/folder in which you want the output files be stored, select the folder, and click on the ‘Add’ button.

---

2 If the data about built-up areas is in vector format, in order to convert the data to raster binary format, users should first convert the feature class or Shapefile to a raster. Users can use the tool ‘Polygon to Raster’ in ArcGIS version 10.3.1 to create the raster data. The second step is to reclassify the output raster file to a binary file. For this purpose, users can use the tool ‘Reclassify’ in ArcGIS version 10.3.1 and change the old values of the raster file to 1 for all built-up area pixels and to 0 for No Data values.
When all the empty bars are filled correctly, click on the ‘OK’ button. Si values calculation tool calls the sivalues.exe tool and computes the Si value for each pixel of built-up area. During this process, a summary report file (step1_working_report.txt) will be created and stored in the Output folder. The output of this process will be stored in the “work_Si.gdb” geodatabase located in the Output folder. This geodatabase will be called by the next tool (Metrics tool) for computation of Dispersion and other metrics of urban sprawl.

4.2 Metrics calculation tool

The purpose of this tool is to calculate the suite of metrics of urban sprawl (e.g., DIS, UP, UD, WUP). The input data for the Metrics calculation tool are:

1. The binary map of built-up areas in raster format (the same raster file that was used in the Si values calculation tool),
2. The geodatabase feature class or the shapefile of the reporting unit(s) which includes two fields in its attribute table: reporting unit(s) identifier and number of inhabitants and jobs, and
3. The output of the first tool (work_Si.gdb) for computation of Dispersion and the other metrics of sprawl (the tool calls this file automatically as long as it is stored properly in the correct directory).

The output of the tool is a shapefile (similar to the shapefile of the reporting unit(s)) that includes all the values of the urban sprawl metrics in its attribute table (see examples in section 5).

4.2.1 How to use Metrics Calculation tool

1. Open the saved ArcMap project in which the USM Toolset has been installed. From Arctoolbox select USM Toolset and click on the second tool [2-Metrics calculation] (Fig. 10).
2. From the ‘Input Raster’ bar skip to the directory folder, select the binary map of built-up areas, and click on the ‘Add’ button.
3. From the ‘Reporting Unit[s] Layer Field’ bar, skip to the directory folder, select the shapefile of reporting unit, and click on the ‘Add’ button.
4. From the ‘Reporting Unit(s) Identifier Field’ drop down menu, select the field in which the ids of the reporting unit(s) is/are stored.

5. From the ‘Number of Inhabitants and Jobs Field’, drop down menu select the field in which the number(s) of inhabitants and jobs is/are stored for the reporting unit(s).

6. From the ‘Output Directory’ bar, skip to the ‘Output’ folder (in which the results will be stored, select the Output folder, and click on the ‘Add’ Button.

7. Finally, from the ‘Output File’ bar skip to the Output folder, type the name of the result file in the ‘Name’ bar (e.g., Results, FinalOutput), select ‘Shapefile’ in the ‘Save as type’ bar, and click on the ‘Save’ button.

![Metrics calculation tool](image)

**Fig. 10:** Metrics calculation tool.

When all the empty bars are filled correctly, click on the ‘OK’ button. The metrics calculation tool uses the input that you have entered to the tool and also the output of the Si values calculation tool and computes the metrics of urban sprawl for the reporting unit(s). During this process, a summary report file (step2_working_report.txt) will be created and stored in the Output folder.

### 4.3 Cleaning tool

The purpose of this tool is to remove all the unnecessary files that have been produced by the Si values and Metrics calculation tools. The only input of the tool is the working directory (i.e., Output folder). The users can decide if they want to delete the ‘work_SI.gdb’ or not. The default option of the tool is ‘No’, i.e., not to delete the geodatabase. If the ‘work_SI.gdb’ has been removed, the calculation of Si values should be repeated for future computations using different reporting unit(s).

#### 4.3.1 How to use the Cleaning tool

1. Open the saved ArcMap project in which the USM Toolset has been installed. From Arctoolbox, select USM Toolset and click on the third tool (3-Cleaning) (Fig. 11).
2. From the ‘Working directory’ bar, skip to the Output folder (or the folder that you want it to be cleaned-up), select the folder and click on the ‘Add’ button.
3. Keep the default option of ‘No’ in ‘Delete Si value geodatabase?’ bar if you want to keep the Si values geodatabase, or select ‘Yes’ if you want to delete this geodatabase along with other files.

---

*This step is optional and its implementation does not make any difference in the final results.*
Fig. 11: Cleaning tool.

When the two empty bars are filled correctly click on the ‘OK’ button. While using this tool, similar to the other two tools of the USM Toolset, a report file (step3_working_report.txt) will be created and stored in the Output folder. The files that exist in the Output folder after running the Cleaning tool (and when keeping the default value for 'Delete Si value geodatabase?') include 'work_Si.gdb', 'step1_working_report.txt', 'step2_working_report.txt', 'step3_working_report.txt' and the final results shapefile. To see the final results, the user should open the shapefile in an ArcMap window, right-click on the shapefile in the 'Table Of Contents' panel, and click on the 'Open Attribute Table' tab. A table will open in ArcMap that includes all the metrics of urban sprawl for the landscape studied.

5. Examples of using the USM Toolset

In this section, you find some theoretic and some real examples of different landscapes and the results of applying the USM Toolset to these landscapes. The files of examples 1-6 and example 9 are available with this tool (on the same website) for users to practice. For each example, users should create two folders: (1) a directory folder (e.g., Directory_ex1) and (2) an Output folder (e.g., Output_ex1). Copy and paste the relevant raster file and shapefile of each example (e.g., example 1) along with the 'sivalues.exe' tool into the directory folder. Then follow the steps described in section 4. Note that when using the second tool (Metrics Calculation Tool), from the 'Reporting Unit(s) Identifier Field' drop down menu, select the 'RU_id' (in the field in which the id of the reporting unit is stored) and from the 'Number of Inhabitants and Jobs Field', drop down menu, select 'inhbjob' (the field in which the number of inhabitants and jobs is stored). Continue with step 3 and you will get the final results that are presented in this user manual.

Example 1: Area of built-up areas = 785,000 m$^2$ (circle with a radius of 500 m), Area of the reporting unit = 3.14 km$^2$, Number of inhabitants and jobs = 2600 people and jobs.
The value of Weighted Urban Proliferation for a landscape of size 3.14 km² and with 785,000 m² of built-up areas and 2,600 inhabitants and jobs is 3.2 UPU/m². Increasing the number of inhabitants and jobs for the same theoretic landscape will decrease the WUP value. See the next example for details.

**Example 2:** Area of the built-up areas = 785,000 m² (circle with a radius of 500 m), Area of the reporting unit = 3.14 km², Number of inhabitants and jobs = 12000 people and jobs.

![Example 2 Image](image.png)

The only difference between the theoretic landscape shown in this example and example 1 is the number of inhabitants and jobs (12,000 versus 2,600 people and jobs). In this example, the higher number of inhabitants and jobs resulted in a higher value of Utilization Density, and therefore, in a lower value of WUP (0.02 UPU/m²).

**Example 3:** Area of built-up areas = 785,000 m² (circle with a radius of 500 m), Area of the reporting unit = 3.14 km², Number of inhabitants and jobs = 0 people and jobs.

![Example 3 Image](image.png)

In this example, the number of inhabitants and jobs is zero and therefore the value of UD is zero indicating that the built-up area is not utilized at all. The value of -1 for LUP indicates an undefined value, because LUP is the result of a division of the area of built-up areas by the number of inhabitants, which is infinity when there are no inhabitants and no jobs.

**Example 4:** Area of built-up areas = 225 m² (1 pixel size of 15 m x 15 m), Area of reporting unit = 3.14 km², Number of inhabitants and jobs = 2 people and jobs.

![Example 4 Image](image.png)

The smallest possible built-up area at any given resolution is one pixel. This will result in very low values of DIS and WUP. The example shown here is for a pixel size of 15 m x 15 m.

**Example 5:** Area of built-up areas = 900 m² (1 pixel size of 30 m x 30 m), Area of reporting unit = 3.14 km², Number of inhabitants and jobs = 5 people and jobs.
Increasing the size of the built-up area results in a higher value of UP and DIS and accordingly, in a higher value of sprawl (0.00044 UPU/m² in this example compared to 0.00002 UPU/m² in example 4).

**Example 6:** Area of built-up areas = 2500 m² (1 pixels size of 50 m x 50 m), Area of reporting unit = 3.14 km², Number of inhabitants and jobs = 14 people and jobs.

Similar to example 5, this example shows that a higher amount of built-up areas results in higher degree of urban sprawl. In this example, the number of inhabitants and jobs was selected proportional to the size of the built-up area to be comparable to example 5 (LUP values in the two examples are very close).

**Example 7:** Area of built-up areas = 123,911,325 m² (in 2011), Area of reporting unit = 246.6 km² (City of Laval, Quebec, Canada), Number of inhabitants and jobs = 510,319 people and jobs; pixel size is 15 m x 15 m (see detailed information in Nazarnia et al. 2016).
Laval is the third largest municipality in the province of Quebec in Canada and the largest suburb of Montreal and one of the highly sprawled urban areas in the Montreal Census Metropolitan Area. This city is geographically separated from the Island of Montreal by the Prairies River. Most of the built-up areas in Laval are located in the centre of the island and along the shore.

**Example 8:** Municipalities located in the west of the Urban Agglomeration of Montreal, Quebec, Canada (western tip of the Island of Montreal). For details on the size of the built-up areas, size of the reporting units, and the number of inhabitants and jobs for each reporting unit, please refer to the table of results (and to Nazarnia et al. 2016). Pixel size is 15 m x 15 m.
Example 9: Area of built-up areas = 27,506,925 m² (in 2011), Area of reporting unit = 74 km² (borough of Beauport in Quebec City, Quebec, Canada, 2011), Number of inhabitants and jobs = 91,569 people and jobs; pixel size is 15 m x 15 m (see detailed information in Nazarnia et al. 2016).

Beauport is a northeastern suburb of Quebec City and is one of the oldest European-founded communities in Canada. Between highly sprawled boroughs of Quebec City, the borough of Beauport is the third-least sprawled area.

We wish you good success with your own urban sprawl analysis!
Acknowledgements

We thank Michael Wenzlaff and Beat Trachtsler for their generous programming support. Michael Wenzlaff programmed the original URSMEC tool (that was used and explained in Jaeger et al. 2008). We also cordially thank Dr. Zachary Patterson for his help with job data for the examples of urban sprawl in Canada. We thank the Swiss Federal Office for the Environment (FOEN, contract no. 06.0111.PZ/M132-2143) and the European Environment Agency (EEA) for their financial support for our project “Urban sprawl in Europe” (Hennig et al. 2015; EEA & FOEN 2016). The development of the USM Toolset has also partly been supported through the project “Controlling urban sprawl – limiting soil consumption” in the context of the Swiss National Research Programme NRP 68 “Soil as a Resource”. In addition, some earlier parts of the work leading to the USM Toolset were funded by the following institutions and foundations: Swiss Federal Institute for Forest, Snow and Landscape Research WSL (Birmensdorf, Switzerland), Paul Schiller Foundation (Lachen, Switzerland), Sophie and Carl Binding Foundation (Basel, Switzerland) and the Bristol Foundation - Ruth und Herbert Uhl-Forschungsstelle für Natur- und Umweltschutz (Schaan, Principality of Liechtenstein).

Appendix A: A few more examples

Example A.1: Area of built-up areas = 900 m² (4 pixels size of 15 m x 15 m), Area of reporting unit = 3.14 km², Number of inhabitants and jobs = 5 people and jobs.

This example is very similar to example 5 in the main text above with the exception of the number of pixels. In example 5, one patch of built-up area with the area of 900 m² is represented by one pixel size of 30 m x 30 m. However, in this example, the same amount of built-up area is represented in a raster file that consists of 4 pixels size of 15 m x 15 m. When the size of the pixels is smaller, the value of Dispersion is slightly smaller because the value of DiS is approximated by the distances between the four pixels (using the distances between the centres of the pixels rather than the distances between all possible pairs of points within each pixel, whereas for each pixel the within-pixel value was calculated for the integral using Mathematica, see Jaeger et al. 2010: Tab. 1), which results in a slightly lower value of WUP.

Example A.2: Area of built-up areas = 2500 m² (4 pixels size of 25 m x 25 m), Area of reporting unit = 3.14 km², Number of inhabitants and jobs = 5 people and jobs.
This example is very similar to example 6 in the main text above with the exception of the number of pixels. In example 6, 2500 m$^2$ of built-up areas were presented by one pixel of 50 m x 50 m. However, in this example the same amount of built-up area is presented in a raster file that consists of 4 pixels size of 25 m x 25 m. When the size of the pixels are smaller the value of Dispersion is slightly smaller for the same reason as mentioned above.

**Example A.3:** Area of built-up areas = 900 m$^2$ (4 pixels size of 15 m x 15 m). Area of reporting unit = 3.14 km$^2$.
Number of inhabitants and jobs = 5 people and jobs.

This example should be compared with example A.1. In example A.1, 900 m$^2$ of built-up area are presented in 4 pixels size of 15 m x 15 m that are distributed in the most compact way (side by side without any distance/empty space between them) in the landscape. In contrast, in this example, the four pixels (size of 15 m x 15 m) are located at a certain distance between them (90 m from edge to edge, or 105 m between the centers of the pixels). This theoretic example shows that the metric of Dispersion depends very much on the relative spatial arrangement of the built-up areas in the landscape. The value of DIS in this example is 11.57 UPU/m$^2$, whereas in example A.1, the DIS value is 4.42 UPU/m$^2$. Accordingly, WUP in this example is 0.00113 UPU/m$^2$, which is also higher than WUP in example A.1.

It very useful to compare this value with a calculation done by hand because this procedure illustrates the use of the formulas by the tool. The most convenient way of calculating DIS is based on the $S_i$ values (for each cell $i$, $i = 1, 2, 3, \ldots$ to $n$), according to the following formulas (Jaeger et al. 2010b: 429-431 and 437-438):

$$S_i = \frac{1}{n_i} \left( \sum_{k=1}^{n_i} \left( \sqrt{\frac{2 \cdot d_k}{T \cdot m} + 1} \right) + WCC(b) \right),$$

where $n_i$ is the number of built-up cells within the HP of cell $i$, including the cell $i$ itself, $d_k$ is the distance between (the centers of) cell $i$ and cell $j$, and $WCC(b)$ is the within-cell contribution to the value of DIS (and to the other metrics), and $b$ is the cell width (in m). For example, when there is only cell $i$ and no other cell within its HP, then $n_i = 1$ and $S_i = WCC(b)$ (since $d_k = 0$). For any chosen reporting unit, DIS can then be calculated based on the $S_i$ values of the cells located within the reporting unit:

$$DIS(b) = \frac{1}{n} \sum_{i=1}^{n} S_i \frac{UPU}{m^2}.$$
where \( n \) is the total number of built-up cells in the reporting unit. In example A.3, this results in:

\[
\begin{align*}
    n_i &= 4 \quad \text{and} \quad n = 4, \\
    \text{WCC (for 15 m x 15 m cell)} &= 2.961, \\
    S_i &= 0.25 \cdot (2 \cdot (\sqrt{2 \cdot 105 + 1} - 1) + (\sqrt{2 \cdot \sqrt{2} \cdot 105 + 1} - 1) + 2.961) \\
    &= 0.25 \cdot (2 \cdot 13.5258 + 16.2622 + 2.961) \\
    &= 0.25 \cdot 46.275 = 11.5687, \text{ and} \\
    \text{DIS} &= 0.25 \cdot (4 \cdot 11.5687) \text{ UPU/m}^2 = 11.5687 \text{ UPU/m}^2.
\end{align*}
\]

Accordingly, \( UP = PBA \cdot \text{DIS} = 0.0002865 \cdot 11.5687 \text{ UPU/m}^2 = 0.00331 \text{ UPU/m}^2 \). This corresponds to the values of DIS and UP provided by the USM Toolset.

A similar calculation can be done for example A.1, using a distance of 15 m between the centers of the cells.

**References**


